HELP

$a \in \mathbb{R}$ this just means a is a real number.

Let's start with the original limit.

$$\lim_{n\to\infty} \left(\frac{n^2 + a \cdot n + 1}{n^2 + 3 \cdot n - 2}\right)^n = \lim_{n\to\infty} e^{\ln\left(\frac{n^2 + a \cdot n + 1}{n^2 + 3 \cdot n - 2}\right)^n}$$
$$= e^{\lim\limits_{n\to\infty} \ln\left(\frac{n^2 + a \cdot n + 1}{n^2 + 3 \cdot n - 2}\right)^n}$$

Now we will focus on just the exponent of e. Keep in mind that we will be raising e to our result.

$$\lim_{n\to\infty} \ln\left(\frac{n^2 + a \cdot n + 1}{n^2 + 3 \cdot n - 2}\right)^n = \lim_{n\to\infty} n \cdot \ln\left(\frac{n^2 + a \cdot n + 1}{n^2 + 3 \cdot n - 2}\right)$$
$$= \lim_{n\to\infty} n \cdot [\ln(n^2 + a \cdot n + 1) - \ln(n^2 + 3 \cdot n - 2)]$$
$$= \lim_{n\to\infty} \frac{\ln(n^2 + a \cdot n + 1) - \ln(n^2 + 3 \cdot n - 2)}{\frac{1}{n}}$$

now since we have a limit with 0 over 0 we can apply L'Hopital's rule

$$= \lim_{n\to\infty} \frac{\frac{2 \cdot n + a}{n^2 + a \cdot n + 1} - \frac{2 \cdot n + 3}{n^2 + 3 \cdot n - 2}}{-n^{-2}}$$
$$= \lim_{n\to\infty} \left(\frac{2 \cdot n^3 + a \cdot n^2}{n^2 + a \cdot n + 1} - \frac{2 \cdot n^3 + 3 \cdot n^2}{n^2 + 3 \cdot n - 2}\right)$$
$$= \lim_{n\to\infty} \left(\frac{2 \cdot n^3}{n^2 + a \cdot n + 1} - \frac{2 \cdot n^3}{n^2 + 3 \cdot n - 2} + \frac{a \cdot n^2}{n^2 + a \cdot n + 1} - \frac{3 \cdot n^2}{n^2 + 3 \cdot n - 2}\right)$$

The part

$$\lim_{n\to\infty} \left(\frac{2 \cdot n^3}{n^2 + a \cdot n + 1} - \frac{2 \cdot n^3}{n^2 + 3 \cdot n - 2}\right)$$

will cancel out (I'll leave that to you). And that just leaves us with

$$\lim_{n\to\infty}\left(\frac{a\cdot n^2}{n^2+a\cdot n+1}-\frac{3\cdot n^2}{n^2+3\cdot n-2}\right)=\lim_{n\to\infty}\left(\frac{a}{1+\frac{a}{n}+\frac{1}{n^2}}-\frac{3}{1+\frac{3}{n}-\frac{2}{n^2}}\right)=a-3$$

since this was an exponent of e we get

$$\lim_{n\to\infty}\left(\frac{n^2+a\cdot n+1}{n^2+3\cdot n-2}\right)^n=e^{a-3}$$

and going back to the original problem we get

$$\lim_{n\to\infty}\left(\frac{n^2+a\cdot n+1}{n^2+3\cdot n-2}\right)^n=e^{a-3}=e$$

so solving for a we have $a=4$.