## 550.100 Introduction to Applied Mathematics and Statistics Problem on Wavelets and Signal Processing

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Assigned on 10/18/11 (Due 10/25/11)

Recall that a function  $H: \mathbb{Z} \to \mathbb{R}$  defined on integers needs to satisfy

- $\sum_{k \in \mathbb{Z}} H(k) = 2$
- $\sum_{k \in \mathbb{Z}} H(k-2l)H(k) = 2\delta_{l,0}$
- $\sum_{k \in \mathbb{Z}} H(k)(-1)^k = 0$

in order for it to be used for constructing wavelets via multi-resolution analysis (MRA).

(a) Let  $\phi : \mathbb{R} \to \mathbb{R}$  be the function defined by

$$\phi(t) := \begin{cases} 1, & \text{if } 0 \le t < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find a function  $H_{\phi} : \mathbb{Z} \to \mathbb{R}$  such that

$$\phi(t) = \sum_{k \in \mathbb{Z}} H_{\phi}(k)\phi(2t - k).$$

- (b) Show that  $H_{\phi}$  found in part (a) satisfies the above three conditions.
- (c) Let  $\varphi : \mathbb{R} \to \mathbb{R}$  be the function defined by

$$\varphi(t) := \begin{cases} t+1, & \text{if } -1 \le t \le 0, \\ 1-t, & \text{if } 0 < t \le 1, \\ 0, & \text{if } |t| > 1. \end{cases}$$

Find a function  $H_{\varphi}: \mathbb{Z} \to \mathbb{R}$  such that

$$\varphi(t) = \sum_{k \in \mathbb{Z}} H_{\varphi}(k)\varphi(2t - k).$$

(d) Determine whether  $H_{\varphi}$  found in part (c) satisfies the above three conditions or not.