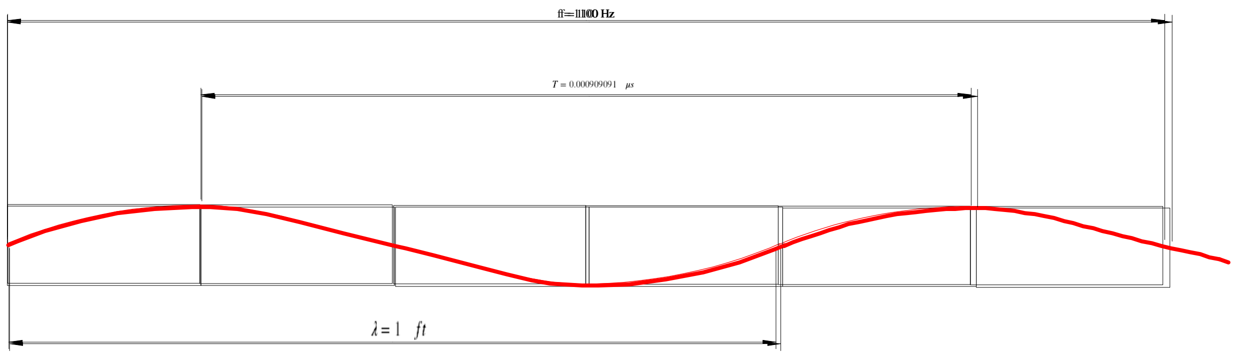
Fourier analysis shows all wave-forms are a combination of sine waves. So I'm using a sine wave.

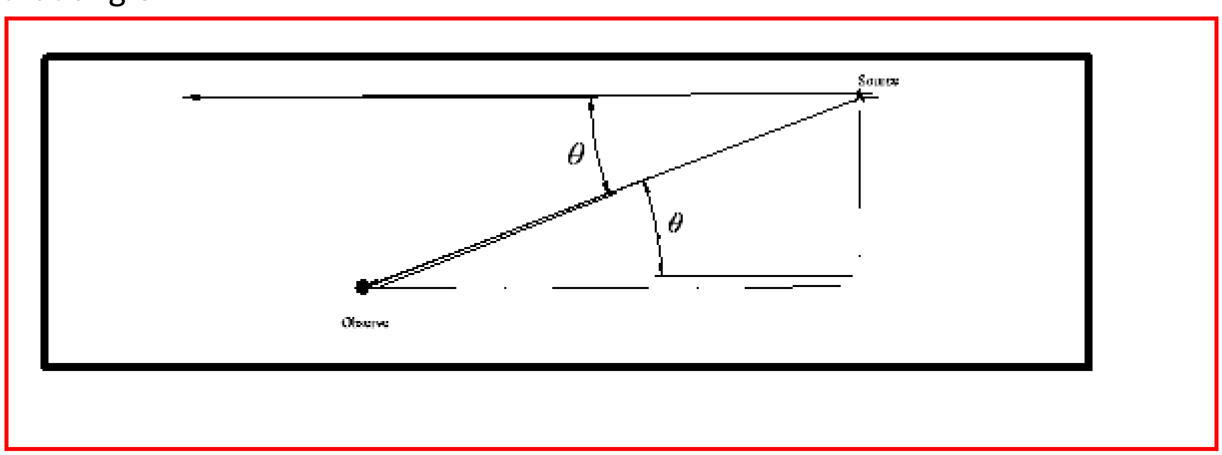


By definition f = 1 / T. T is the period of the sine wave, between consecutive peaks or valleys. Here I indicate T as between peaks. The above picture is f = 1100 Hz, T = 0.009 s and = 1 ft.

"T" is the distance the wave-front, of the wave, travels with velocity "v", distance = velocity * time. So

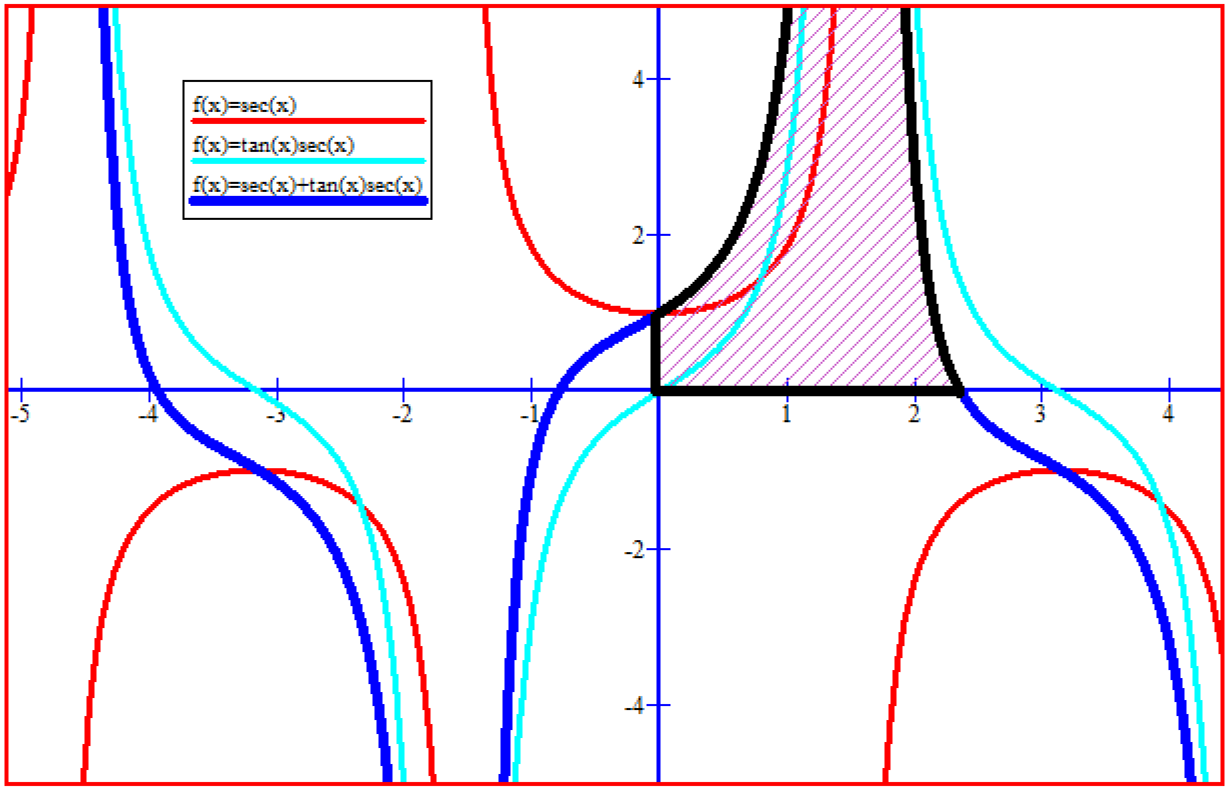
T = λ / v. By substitution --- f = v / .

Alternate interior angles of a line traversing parallel lines are equal. Let "θ" be that angle.



The velocity vector to the observer, from the source, is $v_o = v_s \sec(\theta)$. Thus,

$f_o = (v_s / \lambda) \sec(\theta)$, and $df_o = (v_s / \lambda) \tan(\theta) \sec(\theta)\, d\theta \quad 0 < \theta < \pi$ radians.



The shaded area is relative to the boundary conditions for this problem. The wide blue line is the locus of the instantaneous observed frequencies where $0 < \theta < \Pi$ radians.

The interpretation of the shaded area is: As θ approaches $\frac{\Pi}{2}$ the observed frequency increases above the base observed frequency beyond the hearing range. When θ approaches ∏ the observed frequency decreases below the hearing range. The base observed frequency is:

$f_o(\tan^{-1}(\theta)) = (V_s / \lambda) * (\sec(\tan^{-1}(\theta)))$ where θ is the angle from the souce to the observer when the source is at rest. λ is the constant wave length of the source and V_s is the constant horizontal velocity of the source parallel to the x-axis.

Another way of looking at this problem is that an electronic detector is at the observer and detects the sine wave peaks. The time (T) between peaks is the period which is f=1/T. As the source approaches the observer the the velocity along the vector toward the observer varies and the frequency increases. When the source passes the observer the frequency decreases.

Therefore; if the source is not moving there is no frequency shift.